

# Out-of-plane equilibrium spin current in a quasi-two-dimensional electron gas under in-plane magnetic field

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Equilibrium spin-current is calculated in a quasi-two-dimensional electron gas with finite thickness under in-plane magnetic field and in the presence of Rashba- and Dresselhaus spin-orbit interactions. The transverse confinement is modeled by means of a parabolic potential. An orbital effect of the in-plane magnetic field is shown to mix a transverse quantized spin-up state with nearest-neighbor spin-down states. The out-of-plane component of the equilibrium spin current appears to be not zero in the presence of an in-plane magnetic field, provided at least two transverse-quantized levels are filled. In the absence of the magnetic field the obtained results coincide with the well-known results, yielding cubic dependence of the equilibrium spin current on the spin-orbit coupling constants. The persistent spin-current vanishes in the absence of the magnetic field if Rashba- and Dresselhaus spin-orbit coefficients,  $\alpha$  and  $\beta$ , are equal each other. In-plane magnetic field destroys this symmetry, and accumulates a finite spin-current as  $\alpha \rightarrow \beta$ . Magnetic field is shown to change strongly the equilibrium current of the in-plane spin components, and gives new contributions to the cubic-dependent on spin-orbit constants terms. These new terms depend linearly on the spin-orbit constants.

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## I. INTRODUCTION

A central goal of spintronics research is an achievement of an electron spin manipulation by means of an external electric field [1] instead of a magnetic field, which is widely used now in semiconducting devices for enhancement of an information processing speed. An electric field controlled spin-orbital coupling is a promising tool [2] in realization of futuristic spin transport devices. During the last ten years, there has been impressing progress in both experimental and theoretical understanding of the spin dynamics in quantum wells on the base of, particularly, narrow-gap semiconductors with high g-factor, and metal-oxide-semiconductor field-effect-transistor (MOSFET) structures. Although the dimensionless spin-orbit (SO) coupling parameter in vacuum is as small as  $E_F/(m_0c^2) \sim 10^{-6}$ , where  $E_F \sim 1\text{eV}$  is the Fermi energy of an electron and  $m_0c^2 \sim 1\text{MeV}$  is the Dirac gap, the large value of the SO coupling energy, comparable with the Fermi energy, can be ensured by large potential gradient on the semiconductor/insulator interface of these structures in the presence of macroscopic structural inversion asymmetry (SIA). Indeed, the gate potential applied across the substrate in MOSFET results in inhomogeneous space charge distribution near the semiconductor/insulator interface. The nonuniform macroscopic potential, confining the electrons near the interface, varies over a wide range,  $\sim 10 \div 1000\text{ nm}$ , with larger potential gradient, which originates so-called Rashba SO interaction [3, 4]. On the other hand, higher value of the SO coupling is achieved by choosing special semiconducting materials with a bulk inversion asymmetry (BIA) in their crystalline structure, where the gradi-

ent of the crystal potential is large. Most prominent semiconducting compounds have either zinc-blende structure, like GaAs and most of III-V compounds, or wurtzite structure in II-VI compounds with BIA. Lack of the bulk inversion symmetry in these compounds was shown by Dresselhaus [5] to originate another macroscopic SO interaction.

Effects of both Rashba- and Dresselhaus-SO couplings to the physical properties of two-dimensional (2D) electron gas are not trivial even in the absence of an external magnetic field. Existence of a SIA or a BIA in a disordered 2D system changes reversely the sign of the phase-coherent localization correction to the conductivity [6–8], driving the system from a weak localization regime into an antilocalization one. Rashba- and Dresselhaus SO interactions equally and independently contribute to the weak antilocalization correction. Contributions of Rashba and Dresselhaus SO interactions to the D'yakonov-Perel's spin relaxation rate [9] were shown to be also additive [10]. On the other hand, the anisotropic contribution to the conductivity tensor [11] in the presence of both Rashba and Dresselhaus SO terms, the absence of spin polarization and suppression of spin accumulation especially at the equal values of the coupling constants  $\alpha = \pm\beta$ , [12–14], restoration of the weak localization regime back at  $\alpha = \pm\beta$  [15–17] manifest an existence of the interference between Rashba and Dresselhaus SO interactions. Although the SO coupling generally breaks the spin rotational symmetry, a new type of SU(2) symmetry appears [18] in the case of  $\alpha = \beta$ , which renders the spin lifetime. In the presence of Rashba and Dresselhaus terms with equal strength, the SO interaction rotates electron spins around a single fixed axis. The

spin along this axis becomes conserved, nevertheless spin aligned in the perpendicular directions undergoes a deterministic rotation depending only on the initial and final points of their trajectory.

Different experimental techniques have been developed recently to control a coupling of spin to the electric field [19–21]. An efficient  $\hat{g}$ -tensor modulation resonance, observed in a parabolic  $Al_xGa_{1-x}As$  quantum well [19] with varying  $Al$  content  $x = x(z)$  across the well, provided an opportunity to manipulate electron spins by means of various electron spin resonance type techniques. An in-plane magnetic field in all of these experiments seems to be rather favorable for getting a pronounced spin resonance. SO interactions in a 2D electronic system produce an effective in-plane field, which results in an drift-driven in-plane spin polarization [22]. An external in-plane magnetic field appears to be not always summed algebraically with SO induced effective field, and can result in the surprising out-of-plane spin polarization [23], which has been observed in a strained  $n - InGaAs$  film [20]. On the other hand, Hanle precession of optically oriented 2D electrons in  $GaAs$  [24] is well described by a total in-plane field, given as a sum of the external- and SO effective fields. All these facts show nontrivial effects of in-plane magnetic field on spin dynamics in quasi-2D systems. Effect of an external magnetic field, aligned in the normal direction to the electron gas, has been studied very well, since the problem can be solved exactly for a non-interacting electron gas in the presence of one of the SO interaction. In the previous activities, the selective coupling of the in-plane magnetic field to the electronic spin degree of freedom in the presence of SO interactions has been used to probe the interplay of Zeeman splitting with the SO coupling. The in-plane magnetic field in non-ideal 2D systems with a finite width, which is particularly relevant for heterojunctions and MOSFET structures, couples also to the orbital motion, and can considerably modify the physics involved. It is therefore important to characterize the various physical effects generated in the presence of a parallel magnetic field, in order to gain a better understanding of the influence of orbital magnetic effects on the physical properties of an electron gas with SO interactions.

Although the electron gas, formed on the semiconductor/insulator interface in the heterojunctions and MOSFET structures, has a finite thickness [25], in the most activities concerning the SO interactions it is taken as a strictly 2D object by neglecting the finite thickness. The thickness of the confined electron gas in these structures is varied in the large interval by the gate potential, applied across the electron gas, from 10 nm in the inversion regime up to 1000 nm in the depletion regime. The charge distribution and the electron gas thickness can be experimentally measured and theoretically estimated with high accuracy by means of the self-consistent solutions of Schrödinger and Poisson equations under the

charge balance condition. Finite thickness of the electron gas was recently suggested by Rashba and Efros [26, 27] in order to study the time-dependent gate voltage manipulation of electron spins in MOSFETs and quantum wells, since the spin response to a perpendicular-to plane electric field can be achieved due to a deviation from strict 2D limit.

In this work, we report on our investigation of both orbital and spin effects of in-plane magnetic field in a quasi-two-dimensional (quasi-2D) electron gas with a finite thickness on the spin precession and splitting in the presence of Rashba and Dresselhaus SO interactions. In this paper we calculate persistent spin current. Note that the model has been considered in our previous paper [28, 29] in order to study the energy spectrum and the Fermi surface under variations of SO coupling constants, the gate electric field, the magnetic field and  $g$ -factor. Generation of a spin flux and its change under external destructive factors is still a controversial issue [30] in spintronics. Generation of a dissipationless transverse spin current or a spin Hall current by a driving electric field  $\mathbf{E}$  was predicted [31, 32] in a clean, infinite and homogeneous structural inversion asymmetric 2D system. Even an arbitrary small concentration of non-magnetic impurities was shown [33–36] to suppress totally the universal value of the spin Hall conductivity peculiar to a clean system. As it was shown by Rashba [37], a spin current in the presence of Rashba SO coupling appears even at equilibrium in the absence of an external electric field, though it does not result in any accumulation of spin. A universal equilibrium spin current was shown [38] to appear as a diamagnetic color current due to a response to an effective Yang-Mills magnetic field produced by SO interactions, which provides an explicit realization of a non-Abelian Landau diamagnetism. The equilibrium spin current in a 2D electron gas with a slightly modulated Rashba parameter was shown [39] to transfer spin from areas where spin is produced to areas where spin is absorbed. It was recently shown [40] that an equilibrium spin current in a 2D system with Rashba SO interactions results in a mechanical torque on a substrate near edge of the medium, which provides an experimental tool to detect the equilibrium spin current. Therefore, it can be concluded that a relation of the equilibrium spin current to spin transport should not be ruled out.

The central result of the paper is an appearance of out-of-plane equilibrium spin current in the quasi-2D electron gas under in-plane magnetic field in the presence of the SO interactions. In the absence of the magnetic field, the average values of the spin currents  $\mathbf{J}^{S_x}$  and  $\mathbf{J}^{S_y}$  are shown to coincide with the well-known results [37, 39, 41, 42] obtained for a strictly 2D electron gas, revealing a cubic dependence on the SO coupling constants. We show that the magnetic field strongly changes the in-plane spin-current components, contributing new terms to them. The new contributions turn to be pro-

portional, in addition to the magnetic field, either to the gate electric field or to Zeeman splitting. The new terms depend linearly on the SO coupling constants in the limiting case if one of the SO coupling constant is vanishingly small. Therefore, these contributions may prevail over the cubic dependent on the SO coupling constants terms. The out-of-plane component  $\mathbf{J}^{S_y} = \{0, 0, J_z^{S_z}\}$  vanishes completely with the magnetic field, and depends quadratically or linearly on the SO coupling constants.

The paper is organized as follows. In Section II of this work we describe explicitly an analytical solution of quantum mechanical problem of one particle, moving in a quasi-2D system with finite thickness under in-plane magnetic field and in the presence of Rashba and Dresselhaus SO interactions by imposing a parabolic confining potential in the transverse direction. We take into account in this work a gate potential too, which produces the SIA and Rashba SO interaction. In Section III we calculate the spin current in equilibrium. Conclusions are given in Section IV. In Appendix we present some routine calculations of the persistent spin current.

## II. ENERGY SPECTRUM IN THE PRESENCE OF AN IN-PLANE MAGNETIC FIELD

We consider a quasi-2D gas of electrons, moving under an external in-plane magnetic field in the presence of both Rashba and Dresselhaus SO interactions and a gate potential. Single particle Hamiltonian of the system in the effective mass approximation can be written as

$$\hat{H} = \frac{\mathbf{P}^2}{2m^*} + \frac{m^*\omega_0^2 z^2}{2} - eE_g z + \hat{H}_{so} + \frac{1}{2}g\mu_B\sigma\mathbf{B} \quad (1)$$

where  $\mathbf{P} = \mathbf{p} - \frac{e}{c}\mathbf{A}$  is an electron momentum in the presence of a vector-potential  $\mathbf{A}$ ,  $m^*$  and  $e$  are the electronic effective mass and charge, respectively;  $E_g$  is a strength of the gate electric field. The second term in Eq. (1) is the confining potential in  $z$ -direction, approximated as a parabola with a frequency  $\omega_0$ , which is a characteristic parameter of the electron gas thickness. This potential does not produce a structural inversion asymmetry and, consequently, SO interaction. Since Rashba SO interaction in the conduction band of a semiconductor is determined by the electric field in the valence band rather than by that in the conduction band [43], the parabolic confinement approximation neglects a small interface contribution to Rashba SO coupling constant. The last term in Eq. (1) is Zeeman splitting energy in the external magnetic field  $\mathbf{B}$  with  $\omega_z\hbar = g\mu_B B/2$ , where  $\mu_B = \frac{e\hbar}{2m_0}$  is the Bohr magneton of a free electron with mass  $m_0$ ,  $g$  is the effective Landé factor, and  $\sigma = \{\sigma_x, \sigma_y, \sigma_z\}$  are the Pauli spin matrices.

Spin-orbital Hamiltonian  $\hat{H}_{so}$  in Eq. (1) contains Rashba [3, 4] term,  $\hat{H}_R$ , due to a macroscopic SIA and

Dresselhaus term [5, 15],  $\hat{H}_D$ , due to a BIA in the crystalline structure. Dresselhaus SO interaction in bulk semiconductors with a zinc-blende crystal symmetry is proportional to the third order of the electron momentum  $\mathbf{P}$

$$\hat{H}_D = \frac{\eta}{\hbar} \sum_i \sigma_i P_i (P_{i+1}^2 - P_{i+2}^2), (i = x, y, z; i+3 \rightarrow i), \quad (2)$$

where  $\eta$  is a characteristic bulk coefficient of the SO splitting. Since the average wave vector in the direction of the quantum confinement  $z$  is large, the terms involving  $p_z^2$  will dominate in Dresselhaus SO coupling for a quasi-2D electron gas with finite thickness. The expression for SO interaction Hamiltonian in MOSFETs and quantum wells of the width  $d$  grown along [001] crystallographic axis reads

$$\hat{H}_{so} = \hat{H}_R + \hat{H}_D = \frac{\alpha}{\hbar}(\sigma_x P_y - \sigma_y P_x) + \frac{\beta}{\hbar}(\sigma_x P_x - \sigma_y P_y), \quad (3)$$

where  $\alpha$  and  $\beta = -\eta\langle p_z^2 \rangle = -\eta(\pi/d)^2$  are the sample dependent parameters of Rashba- and Dresselhaus-SO interactions, correspondingly. Rashba coefficient  $\alpha$  is proportional to the gate electric field. Spin-orbital interaction can be interpreted as an interaction of a spin with randomly oriented, in accordance with the electron wave vector, effective magnetic field, which lies in the plane of the electron gas:

$$\hat{H}_R = \frac{\hbar}{2}\sigma \cdot \boldsymbol{\Omega}_{eff}^R \quad \text{with} \quad \boldsymbol{\Omega}_{eff}^R = \frac{2\alpha}{\hbar^2}(\mathbf{P} \times \hat{\mathbf{z}}), \quad (4)$$

$$\hat{H}_D = \frac{\hbar}{2}\sigma \cdot \boldsymbol{\Omega}_{eff}^D, \quad \text{with}$$

$$\boldsymbol{\Omega}_{eff}^D = \frac{2\eta}{\hbar^2}\{P_x(P_y^2 - \langle P_z^2 \rangle), P_y(\langle P_z^2 \rangle - P_x^2), 0\}. \quad (5)$$

Although the effective magnetic field  $\boldsymbol{\Omega}_{eff}^R$ , corresponding to Rashba term is perpendicular to the 2D wave vector of an electron,  $x$ -component of  $\boldsymbol{\Omega}_{eff}^D$  is in the same direction as  $p_x$  while its  $y$ -component is directed in the opposite to  $p_y$  direction.

An external magnetic field is chosen to be in the plane of the 2D electron gas, along  $x$  axis  $\mathbf{B} = \{B, 0, 0\}$  under the gauge  $\mathbf{A} = \{0, -Bz, 0\}$ . The magnetic field tends to polarize the electron spin in the  $x$ -direction due to the Zeeman effect, and creates an angular momentum due to the orbital motion.

In order to solve Schrödinger equation  $i\hbar \frac{\partial \Psi(x, y, z; t)}{\partial t} = \hat{H}\Psi(x, y, z; t)$  with a spinor  $\Psi(x, y, z; t) = \begin{pmatrix} \Psi_\uparrow \\ \Psi_\downarrow \end{pmatrix}$  one expresses the electron wave functions with spin-up  $\Psi_\uparrow$  and spin-down  $\Psi_\downarrow$  orientations as  $\Psi_{\uparrow, \downarrow}(x, y, z; t) = e^{ik_x x + ik_y y} \psi_{\uparrow, \downarrow}(z, t)$  which yields

$$i\hbar \frac{\partial \psi_{\uparrow}}{\partial t} = \left\{ -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + \frac{m^* \omega^2 z^2}{2} + (k_y \hbar \omega_B - eE_g)z + \frac{\hbar^2 k^2}{2m^*} \right\} \psi_{\uparrow} + [(\alpha + i\beta)(k_y + \omega_B m^* z/\hbar) + (\beta + i\alpha)k_x + \hbar \omega_z] \psi_{\downarrow}; \quad (6)$$

$$i\hbar \frac{\partial \psi_{\downarrow}}{\partial t} = \left\{ -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + \frac{m^* \omega^2 z^2}{2} + (k_y \hbar \omega_B - eE_g)z + \frac{\hbar^2 k^2}{2m^*} \right\} \psi_{\downarrow} + [(\alpha - i\beta)(k_y + \omega_B m^* z/\hbar) + (\beta - i\alpha)k_x + \hbar \omega_z] \psi_{\uparrow}; \quad (7)$$

where  $\omega_B = eB/m^*c$  is the cyclotron frequency,  $\omega = \sqrt{\omega_B^2 + \omega_0^2}$  is the effective frequency, and  $k = \sqrt{k_x^2 + k_y^2}$  is the modulus of a 2D-wave vector. In the stationary case the wave function is chosen as  $\Psi(x, y, z; t) = \exp(-iEt/\hbar) \Psi(x, y, z)$ , where  $E$  is the total energy of an electron. In the absence of SO interactions and Zeeman term, Eqs. (6) and (7) are decoupled and reduced to the oscillator equation with real wave function

$$\left\{ -\frac{\hbar^2}{2m^*} \frac{d^2}{dz^2} + \frac{m^* \omega^2}{2} (z - z_0)^2 - \tilde{E} \right\} \psi^{(0)}(z) = 0, \quad (8)$$

where  $z_0$  is the z-coordinate of a magnetic orbit,  $z_0 = \frac{eE_g - k_y \hbar \omega_B}{m^* \omega^2}$ , and  $\tilde{E} = E - \frac{\hbar^2 k^2}{2m^*} + \frac{(eE_g - \hbar k_y \omega_B)^2}{2m^* \omega^2}$  is the energy spectrum of the quantized orbital,  $\tilde{E}_n = \hbar \omega (n + 1/2)$ , corresponding to the  $n$ th state

$$\psi_n^{(0)}(z) = (\sqrt{\pi} 2^n n!)^{-1/2} e^{-\frac{(z-z_0)^2}{2a_B^2}} H_n \left( \frac{z - z_0}{a_B} \right) \quad (9)$$

with  $H_n(z)$  and  $a_B = \sqrt{\hbar/m^* \omega}$  being the Hermite polynomial and the Bohr radius, correspondingly.

SO interactions in Eqs. (6) and (7) mix the transverse-quantized levels, yielding the complex wave functions

$\psi_{\uparrow}(z)$  and  $\psi_{\downarrow}(z)$ ; they furthermore satisfy the condition  $\psi_{\uparrow, \downarrow} = e^{i\theta} \psi_{\downarrow, \uparrow}^*$ . A coordinate-dependent term,  $\propto \omega_B m^* z$ , in the off-diagonal part of Eqs. (6) and (7) is originated from the orbital magnetic field effect, which links  $n$ th orbital of a spin-up electron with  $(n \pm 1)$ th orbital of a spin-down electron and vice versa. Eqs. (6) and (7) are easily solved in the absence of this spatial-dependent term. Indeed, let us replace  $z$  in the 'mixing' terms of Eqs. (6) and (7) by the coordinate of the magnetic orbital center  $z_0$ , and seek the solution as  $\Psi_n^{(0)}(x, y, z) = e^{ik_x x + ik_y y} \psi_n^{(0)}(z) \begin{pmatrix} A_n \\ B_n \end{pmatrix}$ . We get the following system of equations for  $A_n$  and  $B_n$

$$\begin{aligned} (\tilde{E}/\hbar \omega - n - 1/2) A_n - c_0 B_n &= 0 \\ -c_0^* A_n + (\tilde{E}/\hbar \omega - n - 1/2) B_n &= 0, \end{aligned} \quad (10)$$

where the dimensionless coefficient  $c_0$  is given

$$c_0 = \frac{1}{\hbar \omega} \left[ (\alpha + i\beta) \left( k_y \frac{\omega_0^2}{\omega^2} + \frac{eE_g \omega_B}{\hbar \omega^2} \right) + (i\alpha + \beta) k_x + \omega_z \hbar \right]. \quad (11)$$

The energy spectrum is immediately obtained from Eq.(10)

$$\begin{aligned} E_n^{\pm}(k_x, k_y) &= \frac{\hbar^2 k^2}{2m^*} - \frac{(k_y \hbar \omega_B - eE_g)^2}{2m^* \omega^2} + \omega \hbar (n + 1/2) \pm \left\{ (\alpha^2 + \beta^2) \left[ k_x^2 + \frac{(k_y \omega_0^2 + eE_g \omega_B/\hbar)^2}{\omega^4} \right] + \right. \\ &\quad \left. + 4\alpha\beta k_x \frac{(k_y \omega_0^2 + eE_g \omega_B/\hbar)}{\omega^2} + \omega_z^2 \hbar^2 + 2\omega_z \hbar \left[ \alpha \frac{(k_y \omega_0^2 + eE_g \omega_B/\hbar)}{\omega^2} + \beta k_x \right] \right\}^{1/2}. \end{aligned} \quad (12)$$

The coefficients  $A_n$  and  $B_n$  in the spinor are completely defined from the normalization condition  $|A_n|^2 + |B_n|^2 = 1$  and Eq. (10)

$$A_n = \frac{1}{\sqrt{2}} \quad \text{and} \quad B_n = \pm \frac{|c_0|}{\sqrt{2} c_0}. \quad (13)$$

General solutions of Eqs. (6) and (7) are sought as

linear combinations of  $\psi_n^{(0)}(z)$

$$\psi_{\uparrow}(z) = e^{-\frac{(z-z_0)^2}{2a_B^2}} \sum_{n=0}^{\infty} \frac{a_n}{\sqrt{a_B} \sqrt{\pi} 2^n n!} H_n \left( \frac{z - z_0}{a_B} \right); \quad (14)$$

$$\psi_{\downarrow}(z) = e^{-\frac{(z-z_0)^2}{2a_B^2}} \sum_{n=0}^{\infty} \frac{b_n}{\sqrt{a_B} \sqrt{\pi} 2^n n!} H_n \left( \frac{z - z_0}{a_B} \right), \quad (15)$$

where the coefficients  $a_n$  and  $b_n$  satisfy the normalization condition  $\sum_{n=0}^{\infty} (|a_n|^2 + |b_n|^2) = 1$ . From the condition that Eqs. (6) and (7) are complex conjugate each other,

one gets  $a_n = e^{i\theta} b_n^*$  and  $b_n = e^{i\theta} a_n^*$  with  $\theta$  being a real phase shift. Therefore,  $\sum_n |a_n|^2 = \sum_n |b_n|^2 = \frac{1}{2}$ . It is easy to estimate the average value of the spin operator components  $\mathbf{S} = \{\hat{S}_x, \hat{S}_y, \hat{S}_z\} = \hbar/2\{\sigma_x, \sigma_y, \sigma_z\}$  over the stationary states given by Eqs. (14) and (15)

$$\begin{aligned}\langle \hat{S}_x \rangle_{\mathbf{k}} &= \frac{\hbar}{2} \sum_n (a_n^* b_n + b_n^* a_n), \\ \langle \hat{S}_y \rangle_{\mathbf{k}} &= -i \frac{\hbar}{2} \sum_n (a_n^* b_n - b_n^* a_n), \\ \langle \hat{S}_z \rangle_{\mathbf{k}} &= \frac{\hbar}{2} \sum_n (|a_n|^2 - |b_n|^2) = 0, \\ \langle \hat{S}^+ \rangle_{\mathbf{k}} &= \hbar \sum_n a_n^* b_n, \quad \text{and} \quad \langle \hat{S}^- \rangle_{\mathbf{k}} = \hbar \sum_n b_n^* a_n.\end{aligned}\quad (16)$$

So a spin precesses around the normal to the plane, and  $\langle \hat{S}_z \rangle_{\mathbf{k}}$  averages out to zero, whereas in-plane components of the spin take finite values. The average position of an electron in the confining potential  $\langle z \rangle$  can be calculated by the same way,

$$\langle z \rangle_{\mathbf{k}} = \frac{a_B}{\sqrt{2}} \sum_n \sqrt{n+1} \{ (a_{n+1}^* a_n + a_n^* a_{n+1}) + (b_{n+1}^* b_n + b_n^* b_{n+1}) \} + z_0, \quad (17)$$

which means that an overlap between the neighboring transverse-quantized levels shifts the center of the magnetic orbit of both spin-up and spin-down electrons equally, in addition to the magnetic- and gate electric fields shift  $z_0$ , in  $z$ -direction.

Equations for the coefficients  $a_n$  and  $b_n$  with  $n = 0, 1, 2, 3, \dots$  can be obtained by putting Eqs. (14) and (15) into Eqs. (6) and (7)

$$\left( \frac{\tilde{E}}{\hbar\omega} - n - \frac{1}{2} \right) a_n - c_0 b_n - \sqrt{2n} c_1 b_{n-1} - \sqrt{2(n+1)} c_1 b_{n+1} = 0, \quad (18)$$

$$\left( \frac{\tilde{E}}{\hbar\omega} - n - \frac{1}{2} \right) b_n - c_0^* a_n - \sqrt{2n} c_1^* a_{n-1} - \sqrt{2(n+1)} c_1^* a_{n+1} = 0, \quad (19)$$

where  $c_0$  is defined by Eq. (11), and the coefficient  $c_1$  is given as

$$c_1 = (\alpha + i\beta) \frac{\omega_B}{2\hbar\omega} \sqrt{\frac{m^*}{\omega\hbar}}. \quad (20)$$

Note that  $a_n = b_n = 0$  for  $n < 0$  in Eqs. (18) and (19).

It is easy to see that the approximate equations (10) can be obtained from Eqs.(18) and (19) by neglecting all terms  $\sim c_1$ .

In the absence of the external magnetic field,  $B = 0$ , the expressions for  $c_0$  and  $c_1$ , given by Eqs.(11) and (20),

are simplified

$$c_0 = \frac{1}{\omega_0 \hbar} [i\alpha(k_x - ik_y) + \beta(k_x + ik_y)], \quad \text{and} \quad c_1 = 0, \quad (21)$$

and, as a result, a mixing between the transverse-quantized levels is left off (see, Eqs. (18) and (19)). A simple exact expression for the energy spectrum in the absence of the magnetic field is obtained

$$\begin{aligned}E_n^\pm &= \hbar\omega_0(n + \frac{1}{2}) + \frac{\hbar^2 k^2}{2m^*} - \frac{e^2 E_g^2}{2m^* \omega_0^2} \\ &\quad \pm \sqrt{(\alpha^2 + \beta^2)k^2 + 4\alpha\beta k_x k_y},\end{aligned}\quad (22)$$

which is a particular form of Eq. (12) written at  $B = 0$ , since Eq. (12) is exact in this limit.

By expressing  $b_n$  in Eq. (19) through  $a_n, a_{n\pm 1}$  and substituting into Eq. (18) we get an equation for the vector  $\mathbf{a} = \{a_0, a_1, a_2, \dots\}$ . An equation for the vector  $\mathbf{b} = \{b_0, b_1, b_2, \dots\}$  is obtained by the same way; finally we get:

$$\hat{\mathbf{N}}\mathbf{a} = 0 \quad (23)$$

$$\hat{\mathbf{M}}\mathbf{b} = 0 \quad (24)$$

$\hat{\mathbf{N}}$  and  $\hat{\mathbf{M}}$  are square pentadiagonal matrices of infinite order with non-zero entries  $N_{i,j} \neq 0$  ( $M_{i,j} \neq 0$ ) only if  $|i - j| \leq 2$ , and  $\hat{\mathbf{N}} = (\hat{\mathbf{M}})^*$ . Apart from the non-zero main diagonal  $N_{n,n}$ , the matrix  $\hat{\mathbf{N}}$  contains the first two diagonals,  $N_{n,n\pm 1}$  and  $N_{n,n\pm 2}$ , above and below it, which are given as

$$\begin{aligned}N_{n,n} &= \left( \frac{E}{\hbar\omega} - n - \frac{1}{2} \right) - \frac{|c_0|^2}{\frac{E}{\hbar\omega} - n - \frac{1}{2}} - \\ &\quad - \frac{2n|c_1|^2}{\frac{E}{\hbar\omega} - n + \frac{1}{2}} - \frac{2(n+1)|c_1|^2}{\frac{E}{\hbar\omega} - n - \frac{3}{2}};\end{aligned}\quad (25)$$

$$N_{n,n-1} = -\sqrt{2n} \left( \frac{c_1^* c_0}{\frac{E}{\hbar\omega} - n - \frac{1}{2}} + \frac{c_0^* c_1}{\frac{E}{\hbar\omega} - n + \frac{1}{2}} \right); \quad (26)$$

$$N_{n,n+1} = -\sqrt{2(n+1)} \left( \frac{c_1^* c_0}{\frac{E}{\hbar\omega} - n - \frac{1}{2}} + \frac{c_0^* c_1}{\frac{E}{\hbar\omega} - n - \frac{3}{2}} \right); \quad (27)$$

$$N_{n,n-2} = -\frac{2\sqrt{n(n-1)} |c_1|^2}{\frac{E}{\hbar\omega} - n - \frac{1}{2}}; \quad (28)$$

$$N_{n,n+2} = -\frac{2\sqrt{(n+1)(n+2)} |c_1|^2}{\frac{E}{\hbar\omega} - n - \frac{3}{2}}. \quad (29)$$

The energy spectrum has to be found from the secular equation, by equating the determinant of the matrix  $\hat{\mathbf{N}}$  to zero. The infinite pentadiagonal matrix is truncated down to the first  $n$  rows and  $n$  columns, the roots of which can be found by numeric methods, [28, 29].

### III. SPIN CURRENT IN EQUILIBRIUM

The equilibrium spin current in the previous activities [37–42] has been studied for a pure 2D electron gas in the absence of an external magnetic field. This section is addressed to study the spin-current in a quasi-2D electron gas with finite thickness in the presence of an in-plane magnetic field and the gate potential. In order to write the continuity equation for the charge density  $\rho = e|\Psi(z)|^2 = |\psi_\uparrow|^2 + |\psi_\downarrow|^2$  and for the  $\gamma$ -component of the spin density  $S_\gamma = (\hbar/2)(\Psi^\dagger \sigma_\gamma \Psi)$ , the Schrödinger equations (6) and (7) are multiplied to their complex-conjugate components  $\psi_\uparrow^*$  or  $\psi_\downarrow^*$ , which yields

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} &= 0 \\ \frac{\partial S_\gamma}{\partial t} + \nabla \cdot \mathbf{J}^{S_\gamma} &= G_\gamma, \end{aligned} \quad (30)$$

where  $\mathbf{J}$  and  $\mathbf{J}^{S_\gamma}$  are the charge- and the spin-current, correspondingly. A violation of the spin conservation in the system results in an additional source term (torque)  $G_\gamma$  [39] in the spin-balance equation. The components of the charge- and spin currents read

$$\begin{aligned} J_j &= -i \frac{e\hbar}{2m^*} (\Psi^\dagger \nabla_j \Psi - \nabla_j \Psi^\dagger \Psi) - \frac{e\alpha}{\hbar} \Psi^\dagger (\sigma \times \hat{z}_0)_j \Psi \\ &+ \frac{e\beta}{\hbar} \Psi^\dagger \tilde{\sigma}_j \Psi - \frac{e^2}{m^*c} \psi^\dagger A_j \Psi, \end{aligned} \quad (31)$$

and

$$\begin{aligned} J_j^{S_\gamma} &= -i \frac{\hbar^2}{4m^*} [\Psi^\dagger \sigma_\gamma \nabla_j \Psi - \nabla_j \Psi^\dagger \sigma_\gamma \Psi] - \\ &- \frac{\alpha}{4} \{ \Psi^\dagger [\sigma_\gamma (\sigma \times \hat{z}_0)_j + (\sigma \times \hat{z}_0)_j \sigma_\gamma] \Psi \} + \\ &+ \frac{\beta}{4} \{ \Psi^\dagger [\tilde{\sigma}_j \sigma_\gamma + \sigma_\gamma \tilde{\sigma}_j] \Psi \} - \frac{e\hbar}{2m^*c} A_j (\Psi^\dagger \sigma_\gamma \Psi). \end{aligned} \quad (32)$$

where  $\tilde{\sigma} = \{\sigma_x, -\sigma_y, 0\}$ ,  $\hat{z}_0$  is a unit vector, normal to the electron gas plane, and  $A_j$  is  $j$ th component of the vector-potential. It is easy to see that the structure of the contributions coming from Rashba and Dresselhaus terms is similar to the Hamiltonian form given by Eqs. (4) and (5). The magnetic field has a contribution to the charge current as well as to the spin current, which consists with contribution to the charge current [44] obtained by means of Hamilton method in the absence of the SO interactions.

The equilibrium spin- and charge currents at  $T=0$  are found by averaging Eqs. (31) and (32) over the stationary states, given by the wave functions (14) and (15), and by integrating over all occupied states, which yields for the

charge current

$$\langle J_x \rangle = \frac{e\hbar}{m^*} \sum_{n=0}^{n_m} \int \frac{d^2k}{(2\pi)^2} k_x - \frac{e\alpha}{\hbar} \langle \sigma_y \rangle + \frac{e\beta}{\hbar} \langle \sigma_x \rangle = e \sum_{n=0}^{n_m} \int \frac{d^2k}{(2\pi)^2} \left\{ \frac{\hbar k_x}{m^*} + \frac{\beta + i\alpha}{\hbar} a_n^* b_n + \frac{\beta - i\alpha}{\hbar} b_n^* a_n \right\}, \quad (33)$$

$$\begin{aligned} \langle J_y \rangle &= e \left\{ \frac{\hbar}{m^*} \sum_{n=0}^{n_m} \int \frac{d^2k}{(2\pi)^2} k_y + \frac{\alpha}{\hbar} \langle \sigma_x \rangle - \frac{\beta}{\hbar} \langle \sigma_y \rangle + \omega_B \langle z \rangle \right\} \\ &= e \sum_{n=0}^{n_m} \int \frac{d^2k}{(2\pi)^2} \left\{ \frac{\hbar k_y}{m^*} + \frac{\alpha + i\beta}{\hbar} a_n^* b_n + \frac{\alpha - i\beta}{\hbar} b_n^* a_n + \right. \\ &\quad \left. + \omega_B z_0 + \omega_B a_B \sqrt{2(n+1)} (a_{n+1}^* a_n + b_{n+1}^* b_n) \right\}, \end{aligned} \quad (34)$$

$$\langle J_z \rangle = 0 \quad (35)$$

Note that the equations (33)-(35) for the charge current components can be obtained according to  $\langle J_i \rangle = e \langle v_i \rangle$  as well by using the Heisenberg equation of motion  $v_i = \frac{dr_i}{dt} = \frac{i}{\hbar} [\hat{H}, r_i]$ .

The average values of the spin current components read as

$$\begin{aligned} \langle J_x^{S_j} \rangle &= \sum_{n=0}^{n_m} \int \frac{d^2k}{(2\pi)^2} (a_n^* b_n^*) \left( \frac{\hbar k_x}{m^*} \hat{S}_j + \frac{\alpha}{2} \epsilon_{jx} + \frac{\beta}{2} \epsilon_{jy} \right) \begin{pmatrix} a_n \\ b_n \end{pmatrix} \\ &= \int \frac{d^2k}{(2\pi)^2} \left\{ \frac{\hbar^2 k_x}{2m^*} \langle \sigma_j \rangle_{\mathbf{k}, n_m} + \frac{\alpha}{2} \epsilon_{jx} + \frac{\beta}{2} \epsilon_{jy} \right\}, \end{aligned} \quad (36)$$

$$\begin{aligned} \langle J_y^{S_j} \rangle &= \sum_{n=0}^{n_m} \int \frac{d^2k}{(2\pi)^2} \left\{ (a_n^* b_n^*) \left( \frac{\hbar k_y}{m^*} \frac{\omega_0^2}{\omega^2} + \frac{eE_g \omega_B}{m^* \omega^2} \right) \hat{S}_j + \right. \\ &\quad \left. + \frac{\alpha}{2} \epsilon_{jy} + \frac{\beta}{2} \epsilon_{jx} + \omega_B a_B \sqrt{2(n+1)} (a_{n+1}^* b_{n+1}^*) \hat{S}_j \right\} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \\ &= \int \frac{d^2k}{(2\pi)^2} \left\{ \left( \frac{\hbar^2 k_y}{m^*} \frac{\omega_0^2}{\omega^2} + \frac{eE_g \omega_B \hbar}{m^* \omega^2} \right) \langle \sigma_j \rangle_{\mathbf{k}, n_m} + \right. \\ &\quad \left. + \frac{\alpha}{2} \epsilon_{jy} + \frac{\beta}{2} \epsilon_{jx} + \frac{a_B}{2} \omega_B \hbar \langle \sigma_j \rangle_{\mathbf{k}, n_m}^{off} \right\} \end{aligned} \quad (37)$$

and

$$\begin{aligned} \langle J_{k_z}^{S_z} \rangle &= i \frac{\hbar}{m^* a_B} \sum_{n=0}^{n_m} \int \frac{d^2k}{(2\pi)^2} \sqrt{2(n+1)} \times \\ &\quad (a_{n+1}^* b_{n+1}^*) \hat{S}_z \begin{pmatrix} a_n \\ b_n \end{pmatrix} = i \frac{\hbar^2}{2m^* a_B} \sum_{n_m} \int \frac{d^2k}{(2\pi)^2} \langle \sigma_z \rangle_{\mathbf{k}, n_m} \end{aligned} \quad (38)$$

where  $j = x, y$  and  $\langle J_z^{S_x} \rangle = \langle J_z^{S_y} \rangle = 0$ , also  $\langle J_x^{S_z} \rangle = \langle J_y^{S_z} \rangle = 0$ ;  $\epsilon_{ij}$  is a 2D antisymmetric tensor with components  $\epsilon_{xy} = -\epsilon_{yx} = 1$ . The expressions for  $\langle \sigma_x \rangle_{\mathbf{k}, n_m}$ ,  $\langle \sigma_y \rangle_{\mathbf{k}, n_m}$ ,  $\langle \sigma_z \rangle_{\mathbf{k}, n_m}$  and their evident momentum dependences are calculated in Appendix.

It is evident that the normal to the electron gas component of  $\langle \mathbf{J}^{S_z} \rangle$  in Eq. (38) arises exclusively due to the

transverse confinement and the in-plane magnetic field. Nevertheless this term does not accumulate a spin, an electron tunneling from  $n$ th to  $(n+1)$ th level is accompanied, according to Eq. (38), by reverse flow from  $(n+1)$ th to  $n$ th level.

The spin-continuity equation (30) contains the source-term  $G_\gamma$  due to a violation of the spin conservation, the components of which are given by the following expressions

$$G_j = -i\frac{\alpha}{2}\{\Psi^\dagger[\sigma \times (\hat{z}_0 \times \nabla)]_j\Psi - [(\nabla \times \hat{z}_0) \times \sigma]_j\Psi^\dagger\Psi\} - i\frac{\beta}{2}\{\Psi^\dagger(\tilde{\nabla} \times \sigma)_j\Psi - (\tilde{\nabla} \times \sigma)_j\Psi^\dagger\Psi\} + \beta\frac{e}{c}\Psi^\dagger(\mathbf{A} \times \sigma)\Psi + \frac{1}{2}g\mu_B\Psi^\dagger(\mathbf{B} \times \sigma)\Psi + \alpha\frac{e}{c}\Psi^\dagger((\hat{z}_0 \times \mathbf{A}) \times \sigma)\Psi, \quad (39)$$

where  $\tilde{\nabla} = \{\nabla_x, -\nabla_y, 0\}$  and  $\tilde{\sigma} = \{\sigma_x, -\sigma_y, 0\}$ . The averaging of x- and y-components of the torque over the quantum-mechanic states gives

$$\langle G_x \rangle = 0, \quad \text{and} \quad \langle G_y \rangle = 0, \quad (40)$$

which is in consistence with a result in strictly 2D system [39] in the absence of an external magnetic field. Nevertheless z-component of the torque is not averaged to zero:

$$\begin{aligned} \langle G_z \rangle = & \sum_{n=0}^{n_m} \int \frac{d^2k}{(2\pi)^2} \left\{ (\alpha k_x + \beta k_y + \frac{\beta}{\hbar} z_0 \omega_B m^*) (a_n^* b_n + b_n^* a_n) - i(\alpha k_y + \beta k_x + \frac{1}{2}g\mu_B B + \frac{\alpha}{\hbar} z_0 \omega_B m^*) (a_n^* b_n - b_n^* a_n) - i\frac{\alpha}{\hbar} a_B \omega_B m^* \sqrt{2(n+1)} (a_{n+1}^* b_n - b_{n+1}^* a_n) + \frac{\beta}{\hbar} a_B \omega_B m^* \sqrt{2(n+1)} (a_{n+1}^* b_n + b_{n+1}^* a_n) \right\}. \end{aligned} \quad (41)$$

The integration at  $T = 0$  is taken over each momentum component  $i = x, y$  and for both spin branches in each transverse-quantized subband up to the Fermi level,  $-K_{n_m, \pm}^i \leq k_i \leq K_{n_m, \pm}^i$ . In order to calculate the average values of the charge- and spin current components, given by Eqs. (35)-(30), evident expressions of  $a_n$  and  $b_n$  are required. The Fermi level is assumed to be set between  $n_m$  and  $n_m + 1$  subbands, so that all levels up to  $\{\mathbf{K}_m, n_m\}$  are occupied with  $a_n \neq 0$ ,  $b_n \neq 0$  for  $n \leq n_m$  and  $a_n = b_n = 0$  for  $n > n_m$ . A simplest case, which takes into account the inter-subband mixing due to an interference between the SO interactions and in-plane magnetic field, is  $n_m = 1$ . In this case equations (18) and (19) are simplified to the form, given by Eq. (79) in Appendix. The analytical expressions for the energy spectrum in the first and second transverse-quantized subbands are given according to Eqs. (82)-(85)

$$E_\pm^{(n)} = \frac{\hbar^2 k^2}{2m^*} - \frac{(eE_g - \hbar\omega_B k_y)^2}{2m^* \omega^2} + \hbar\omega + \lambda_n \frac{\hbar\omega}{2} \times \sqrt{1 + 4|c_0|^2 + 8|c_1|^2 \mp 4\sqrt{|c_0|^2 + 2(c_0^* c_1 + c_0 c_1^*)^2}}, \quad (42)$$

where  $\lambda_n$  for  $n = 0, 1$  indicates the sub-band index with  $\lambda_0 = -$  and  $\lambda_1 = +$ , and the sign  $\pm$  shows the spin-branch index. In-plane momentum dependence of the energy spectrum, Eq. (42), is determined by the terms  $|c_0|^2$  and  $(c_0^* c_1 + c_0 c_1^*)^2$

$$|c_0|^2 = \frac{1}{\hbar^2 \omega^2} \left\{ (\alpha^2 + \beta^2) \left[ \left( k_y \frac{\omega_0^2}{\omega^2} + \frac{eE_g \omega_B}{\hbar \omega^2} \right)^2 + k_x^2 \right] + 4\alpha\beta k_x \left( k_y \frac{\omega_0^2}{\omega^2} + \frac{eE_g \omega_B}{\hbar \omega^2} \right) + 2\omega_z \hbar \left[ \alpha \left( k_y \frac{\omega_0^2}{\omega^2} + \frac{eE_g \omega_B}{\hbar \omega^2} \right) + \beta k_x \right] \right\}, \quad (43)$$

$$c_0^* c_1 + c_0 c_1^* = \frac{\omega_B}{(\omega \hbar)^2} \sqrt{\frac{m^*}{\omega \hbar}} \left\{ (\alpha^2 + \beta^2) \left( k_y \frac{\omega_0^2}{\omega^2} + \frac{eE_g \omega_B}{\hbar \omega^2} \right) + 2\alpha\beta k_x + \alpha\omega_z \hbar \right\} \quad (44)$$

So,  $|c_0|^2 \sim O(\alpha^2, \beta^2)$ , whereas  $(c_0^* c_1 + c_0 c_1^*)^2 \sim O(\alpha^4, \beta^4, \alpha^2 \beta^2)$  in the absence of Zeeman splitting. Therefore, expansion of Eq. (42) over small SO coupling constants up to quadratic in  $\alpha, \beta$  terms yields

$$E_\pm^{(0)} \approx \frac{\hbar^2 k^2}{2m^*} - \frac{(eE_g - \hbar\omega_B k_y)^2}{2m^* \omega^2} + \frac{1}{2}\hbar\omega \pm |c_0|\hbar\omega; \quad (45)$$

$$E_\pm^{(1)} \approx \frac{\hbar^2 k^2}{2m^*} - \frac{(eE_g - \hbar\omega_B k_y)^2}{2m^* \omega^2} + \frac{3}{2}\hbar\omega \mp |c_0|\hbar\omega. \quad (46)$$

The limit of integration over the occupied states can be found by fixing the Fermi energy  $E_F$  in Eq. (42) and solving this equation for the momentum. It is necessary to note that an interference between the gate electric field and the orbital effect of the in-plane magnetic field shifts the Fermi surface along  $k_y$  axis (see, Eqs. (45), (46) and (43)). Furthermore, Zeeman splitting makes the energy spectra asymmetric along both  $k_x$  and  $k_y$  axes. Therefore, the integrations over  $k_x$  and  $k_y$  have to be taken, generally speaking, over asymmetric intervals  $-K_{n_m, \pm}^x \leq k_x \leq K_{n_m, \pm}^x$  and  $-K_{n_m, \pm}^y \leq k_y \leq K_{n_m, \pm}^y$ .

The integration limit is calculated in Appendix by fixing the Fermi energy and transforming the momentum components in Eqs. (45) and (46) into polar coordinates,  $k_x = k \cos \varphi$ ,  $k_y = k \sin \varphi$ . We express Eq. (110) for  $k_{n, \pm}^F$  as  $k_{n, \pm}^F = k_n^F \pm \delta k$ , where

$$k_n^F = \frac{1}{\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi} \left\{ -\frac{eE_g\omega_B}{\hbar\omega^2} \sin \varphi + \left[ \frac{2m^*}{\hbar^2} [E_F - \omega\hbar(n+1/2)] (\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi) + \frac{e^2 E_g^2}{\omega^2 \hbar^2} + \frac{m^{*2}}{\hbar^4} [(\alpha^2 + \beta^2) \left( \cos^2 \varphi + \frac{\omega_0^4}{\omega^4} \sin^2 \varphi \right) + 4\alpha\beta \frac{\omega_0^2}{\omega^2} \sin \varphi \cos \varphi] \right]^{1/2} \right\}, \quad (47)$$

$$\delta k = \frac{m^*}{\hbar^2 (\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi)} \sqrt{(\alpha^2 + \beta^2) \left( \cos^2 \varphi + \frac{\omega_0^4}{\omega^4} \sin^2 \varphi \right) + 4\alpha\beta \frac{\omega_0^2}{\omega^2} \sin \varphi \cos \varphi}. \quad (48)$$

for  $n = 0, 1$ .

The average values of the Pauli spin-matrices  $\langle \sigma_j \rangle_{\mathbf{k}, n_m}$ ,  $j = x, y, z$  are calculated in Appendix. By using the expressions (11), (20), (43) and (99) in Eqs. (94), (95) and (98) in Appendix, one gets an explicit momentum-dependent expressions for the averaged Pauli matrices

$$\langle \sigma_x \rangle_{k_{\pm}, n} = \pm \lambda_n \frac{ak + b}{\sqrt{ck^2 + dk + e}}, \quad (49)$$

$$\langle \sigma_y \rangle_{k_{\pm}, n} = \mp \lambda_n i \frac{\bar{a}k + \bar{b}}{\sqrt{ck^2 + dk + e}}, \quad (50)$$

$$\langle \sigma_z \rangle_{k_{\pm}, n} = \mp \lambda_n i \frac{\tilde{a}k + \tilde{b}}{\sqrt{ck^2 + dk + e}}, \quad (51)$$

where  $k$  is the momentum modulus  $\{k_x, k_y\} = \{k \cos \phi, k \sin \phi\}$  in spherical-polar system and

$$a = \alpha \frac{\omega_0^2}{\omega^2} \sin \phi + \beta \cos \phi, \quad (52)$$

$$b = \alpha \frac{eE_g\omega_B}{\hbar\omega^2} + \omega_z \hbar, \quad (53)$$

$$\bar{a} = \alpha \cos \phi + \beta \frac{\omega_0^2}{\omega^2} \sin \phi, \quad (54)$$

$$\bar{b} = \beta \frac{eE_g\omega_B}{\hbar\omega^2}, \quad (55)$$

$$\tilde{a} = \frac{\omega_B}{\omega \hbar} \sqrt{\frac{m^*}{\omega \hbar}} (\alpha^2 - \beta^2) \cos \varphi, \quad (56)$$

$$\tilde{b} = -\frac{\omega_B}{\omega} \sqrt{\frac{m^*}{\omega \hbar}} \beta \omega_z, \quad (57)$$

$$c = \left( \alpha \frac{\omega_0^2}{\omega^2} \sin \phi + \beta \cos \phi \right)^2 + \left( \beta \frac{\omega_0^2}{\omega^2} \sin \phi + \alpha \cos \phi \right)^2, \quad (58)$$

$$d = 2 \left( \frac{2eE_g\omega_B}{\hbar\omega^2} \right) \left[ \alpha \left( \alpha \frac{\omega_0^2}{\omega^2} \sin \phi + \beta \cos \phi \right) + \beta \left( \beta \frac{\omega_0^2}{\omega^2} \sin \phi + \alpha \cos \phi \right) \right] + 2\omega_z \hbar \left( \alpha \frac{\omega_0^2}{\omega^2} \sin \phi + \beta \cos \phi \right), \quad (59)$$

$$e = \left( \alpha \frac{eE_g\omega_B}{\hbar\omega^2} + \omega_z \hbar \right)^2 + \left( \beta \frac{eE_g\omega_B}{\hbar\omega^2} \right)^2. \quad (60)$$

In-plane magnetic field induces an inter-sub-band coupling terms,  $\langle \sigma_i \rangle_{\mathbf{k}, n_m}^{off}$  with  $i = x, y$ , which give a contribution to the  $y$ -components of the spin current (37). These terms are calculated in Appendix. By neglecting the small terms  $[c_0^{*2}c_1 + c_0^2c_1^* + (c_1^* + c_1)|c_0|^2]$  and  $[c_0^{*2}c_1 - c_0^2c_1^* + (c_1^* - c_1)|c_0|^2]$ , Eqs. (97) and (97) in Appendix are approximated as

$$\begin{aligned} \langle \sigma_x \rangle_{\mathbf{k}, n_m}^{off} &\approx \frac{c_1^* + c_1}{2(\tilde{\epsilon} - 1)} = \lambda_n (c_1^* + c_1) = \\ &= \lambda_n \frac{\alpha \omega_B}{\omega \hbar} \sqrt{\frac{m^*}{\omega \hbar}}, \end{aligned} \quad (61)$$

$$\begin{aligned} \langle \sigma_y \rangle_{\mathbf{k}, n_m}^{off} &\approx -i \frac{c_1^* - c_1}{2(\tilde{\epsilon} - 1)} = -i \lambda_n (c_1^* - c_1) = \\ &= -\lambda_n \frac{\beta \omega_B}{\omega \hbar} \sqrt{\frac{m^*}{\omega \hbar}}. \end{aligned} \quad (62)$$

So, the inter-subband coupling terms  $\langle \sigma_x \rangle_{\mathbf{k}, n_m}^{off}$  and  $\langle \sigma_y \rangle_{\mathbf{k}, n_m}^{off}$  depend only on the sub-band index and do not depend on the spin-branch index.

In order to calculate the equilibrium spin-current components, Eqs. (36)- (38) are integrated firstly over the momentum modulus  $k$  by taking into account Eqs. (49)-(62). Routine calculations yield under this condition the following results for the equilibrium spin-current  $\mathbf{J}^{S_x} = \{J_x^{S_x}, J_y^{S_x}, 0\}$



$$\langle J_x^{S_x} \rangle = \frac{m^* \omega_0}{3\pi \hbar^4 \omega} \beta (\beta^2 - \alpha^2) + \frac{1}{2\pi \beta} \left( \frac{e E_g \omega_B}{\omega_0^2 \hbar} \right)^2 \left\{ \frac{1}{2} (\alpha^2 + 2\beta^2 - |\alpha^2 - \beta^2|) + \frac{1}{8\alpha^2} [(\alpha^2 - \beta^2)^2 - |\alpha^2 - \beta^2|(\alpha^2 + \beta^2)] + \frac{\alpha^2 + \beta^2}{3} \left( 1 - \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} \right) \right\} + \frac{\omega_z}{4\pi \alpha \beta} \frac{e E_g \omega_B}{\omega_0^2} \left\{ \alpha^2 + \beta^2 - |\alpha^2 - \beta^2| + \frac{7\alpha^2}{3} \left( 1 - \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} \right) \right\} + \frac{5\omega_z^2 \hbar^2}{12\pi \beta} \left\{ 1 - \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} - \frac{\alpha^2 + \beta^2 - |\alpha^2 - \beta^2|}{\alpha^2} \right\}, \quad (63)$$

$$\langle J_y^{S_x} \rangle = \frac{m^* \omega_0}{3\pi \hbar^4 \omega} \alpha [(\alpha^2 - \beta^2) + \frac{3\omega_B^2}{\omega_0^2} (\alpha^2 + \beta^2)] + \frac{\alpha m^* \omega_B^2}{\pi \hbar^2 \omega \omega_0} \left( E_F - \frac{3}{2} \omega \hbar \right) - \frac{1}{48\pi \alpha \beta^2} \left( \frac{e E_g \omega_B}{\omega_0^2 \hbar} \right)^2 \left\{ 3\alpha^4 - 17\beta^4 - 62\alpha^2 \beta^2 + 3(3\beta^2 - \alpha^2) |\alpha^2 - \beta^2| - 8\beta^2 \frac{(\alpha^4 - \beta^4)}{|\alpha^2 - \beta^2|} \right\} - \frac{\omega_z}{12\pi \alpha^2 \beta^2} \frac{e E_g \omega_B}{\omega_0^2} \left\{ 3\alpha^4 - 3\beta^4 - 3(\alpha^2 - \beta^2) |\alpha^2 - \beta^2| - 6\alpha^2 \beta^2 \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} \right\} + \frac{5\omega_z^2 \hbar^2}{12\pi \alpha} \left\{ 1 + \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} - \frac{\alpha^2 + \beta^2 - |\alpha^2 - \beta^2|}{\beta^2} \right\}, \quad (64)$$

as well as for  $\mathbf{J}^{S_y} = \{J_x^{S_y}, J_y^{S_y}, 0\}$

$$\langle J_x^{S_y} \rangle = \frac{m^* \omega_0}{3\pi \hbar^4 \omega} \alpha (\beta^2 - \alpha^2) + \frac{1}{48\pi \alpha \beta^2} \left( \frac{e E_g \omega_B}{\omega_0^2 \hbar} \right)^2 \left\{ 3(\alpha^2 + 5\beta^2) |\alpha^2 - \beta^2| - (\alpha^2 + \beta^2)(3\alpha^2 + 23\beta^2) - 8\beta^2 (\alpha^2 + \beta^2) \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} \right\} - \frac{\omega_z}{24\pi \alpha^2 \beta^2} \frac{e E_g \omega_B}{\omega_0^2} \left\{ (\alpha^2 - \beta^2) |\alpha^2 - \beta^2| - (\alpha^4 - \beta^4) + 2\alpha^2 \beta^2 \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} \right\} - \frac{\omega_z^2 \hbar^2}{6\pi \alpha} \left\{ 1 + \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} - \frac{\alpha^2 + \beta^2 - |\alpha^2 - \beta^2|}{\beta^2} \right\}, \quad (65)$$

$$\langle J_y^{S_y} \rangle = \frac{m^* \omega_0}{3\pi \hbar^4 \omega} \beta \left[ (\alpha^2 - \beta^2) - \frac{3\omega_B^2}{\omega_0^2} (\alpha^2 + \beta^2) \right] - \frac{\beta m^* \omega_B^2}{\pi \hbar^2 \omega \omega_0} \left( E_F - \frac{3}{2} \omega \hbar \right) + \frac{1}{48\pi \alpha^2 \beta} \left( \frac{e E_g \omega_B}{\omega_0^2 \hbar} \right)^2 \left\{ 3\alpha^4 + 3\beta^4 - 3(\alpha^2 + \beta^2) |\alpha^2 - \beta^2| - 8\alpha^2 (\alpha^2 + \beta^2) \left( 1 - \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} \right) \right\} + \frac{\omega_z}{24\pi \alpha \beta} \frac{e E_g \omega_B}{\omega_0^2} \left\{ -5\alpha^2 - 3\beta^2 - 3|\alpha^2 - \beta^2| + 2(4\alpha^2 - 3\beta^2) \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} \right\} - \frac{\omega_z^2 \hbar^2}{6\pi \beta} \left\{ 1 - \frac{|\alpha^2 - \beta^2|}{(\alpha^2 - \beta^2)} - \frac{\alpha^2 + \beta^2 - |\alpha^2 - \beta^2|}{\alpha^2} \right\}, \quad (66)$$

The non-zero component of  $S_z$  spin-current,  $\mathbf{J}^{S_z} = \{0, 0, J_z^{S_z}\}$ , induced by the in-plane magnetic field, can be presented as

$$\langle J_z^{S_z} \rangle = \frac{m^* \omega_B (\alpha^2 - \beta^2)}{4\pi \alpha \beta \omega \hbar^2} \left\{ (\alpha^2 + \beta^2 - |\alpha^2 - \beta^2|) \left( \frac{e E_g \omega_B}{\hbar \omega^2} \right) + \alpha \omega_z \hbar \left( 1 - \frac{|\alpha^2 - \beta^2|}{\alpha^2 - \beta^2} \right) \right\} + \frac{m^* \beta \omega_B \omega_z}{\pi \omega_0 \hbar}. \quad (67)$$

The above expressions for the persistent spin current are simplified considerably as the gate voltage and Zeeman splitting vanish,  $E_g = \omega_z = 0$ ,

$$\mathbf{J}^{S_x} = \left\{ \frac{m^* \omega_0}{3\pi \hbar^4 \omega} \beta (\beta^2 - \alpha^2), \frac{m^* \omega_0}{3\pi \hbar^4 \omega} \alpha \left[ (\alpha^2 - \beta^2) + \frac{3\omega_B^2}{\omega_0^2} (\alpha^2 + \beta^2) \right] + \frac{\alpha m^* \omega_B^2}{\pi \hbar^2 \omega \omega_0} \left( E_F - \frac{3}{2} \omega \hbar \right), 0 \right\} \quad (68)$$

$$\mathbf{J}^{S_y} = \left\{ \frac{m^* \omega_0}{3\pi \hbar^4 \omega} \alpha (\beta^2 - \alpha^2), \frac{m^* \omega_0}{3\pi \hbar^4 \omega} \beta \left[ (\alpha^2 - \beta^2) - \frac{3\omega_B^2}{\omega_0^2} (\alpha^2 + \beta^2) \right] - \frac{\beta m^* \omega_B^2}{\pi \hbar^2 \omega \omega_0} \left( E_F - \frac{3}{2} \omega \hbar \right), 0 \right\}; \quad (69)$$

$$\mathbf{J}^{S_z} = \{0, 0, 0\}. \quad (70)$$

Although the spin-current components turn to vanish at  $\alpha = \beta$  in the absence of the magnetic field, the latter destroys this symmetry and yields a finite spin current as  $\alpha \rightarrow \beta$ . Indeed, the expressions (63)-(67) yield the well-known results of Refs. [41, 42] for spin current in a

2D electron gas in the absence of the magnetic field

$$\mathbf{J}^{S_x} = \left\{ \frac{m^*}{3\pi \hbar^4} \beta (\beta^2 - \alpha^2), \frac{m^*}{3\pi \hbar^4} \alpha (\alpha^2 - \beta^2), 0 \right\}; \quad (71)$$

$$\mathbf{J}^{S_y} = \left\{ \frac{m^*}{3\pi \hbar^4} \alpha (\beta^2 - \alpha^2), \frac{m^*}{3\pi \hbar^4} \beta (\alpha^2 - \beta^2), 0 \right\}, \quad (72)$$

$$\mathbf{J}^{S_z} = \{0, 0, 0\}. \quad (73)$$

It is easy to check that the diagonal components of the spin current vanish,  $\langle J_x^{S_x} \rangle = \langle J_y^{S_y} \rangle = \langle J_z^{S_z} \rangle = 0$  as  $\beta \rightarrow 0$

for arbitrary  $\alpha \neq 0$ . Nevertheless

$$\langle J_y^{S_x} \rangle = \frac{m^{*2}\omega_0\alpha^3}{3\pi\hbar^4\omega} \left(1 + \frac{3\omega_B^2}{\omega^2}\right) + \frac{\alpha m^*\omega_B^2}{\pi\hbar^2\omega\omega_0} \left(E_F - \frac{3}{2}\omega\hbar\right) + \frac{29\alpha}{24\pi} \left(\frac{eE_g\omega_B}{\hbar\omega_0^2}\right)^2; \quad (74)$$

$$\langle J_x^{S_y} \rangle = -\frac{m^{*2}\omega_0\alpha^3}{3\pi\hbar^4\omega} - \frac{11\alpha}{24\pi} \left(\frac{eE_g\omega_B}{\hbar\omega_0^2}\right)^2, \quad (75)$$

under these conditions. On the other hand the diagonal spin-current components are nonzero for  $\alpha \rightarrow 0$  and  $\beta \neq 0$

$$\langle \mathbf{J}^{S_x} \rangle = \left\{ \frac{m^{*2}\omega_0\beta^3}{3\pi\hbar^4\omega} + \frac{11\beta}{24\pi} \left(\frac{eE_g\omega_B}{\hbar\omega_0^2}\right)^2; 0; 0 \right\}, \quad (76)$$

$$\langle \mathbf{J}^{S_y} \rangle = \left\{ 0; -\frac{m^{*2}\omega_0\beta^3}{3\pi\hbar^4\omega} \left(1 + \frac{3\omega_B^2}{\omega^2}\right) - \frac{\beta m^*\omega_B^2}{\pi\hbar^2\omega\omega_0} \left(E_F - \frac{3}{2}\omega\hbar\right) - \frac{\beta}{3\pi} \left(\frac{eE_g\omega_B}{\hbar\omega_0^2}\right)^2; 0 \right\}, \quad (77)$$

$$\langle \mathbf{J}^{S_z} \rangle = \left\{ 0; 0; \frac{m^*\beta\omega_B\omega_z}{2\pi\omega_0\hbar} \right\}. \quad (78)$$

New contributions to the pure 2D ( $\sim \alpha^3, \beta^3$ ) spin-current expressions in Eqs. (63)-(66) are caused by electron-transfer mechanism between nearest-neighbor transverse-quantized sub-bands, induced by the in-plane magnetic field, and they vanish, consequently, with magnetic field. These terms are proportional either to the gate electric field or to Zeeman splitting too. Indeed, the gate electric field changes, on the one hand, the electronic energy spectrum, and shifts, on the other hand, the center of an magnetic orbit  $z_0$  along  $z$ -axis. The coefficient  $c_0$ , which has a physical meaning of probability amplitude for an electron transition from one spin-polarized branch to other one in the same transverse-quantized subband  $n$ , according to Eqs. (18) and (19), parametrically depends on  $E_g$  and  $\omega_z$ . The new terms in the spin-current expressions depend linearly on the SO coupling constants in the simplest case if one of the SO coupling constant is zero. Therefore, these terms may dominate over the pure 2D terms for some values of  $\alpha$ ,  $\beta$ ,  $E_g$  and magnetic field.

#### IV. CONCLUSIONS

In this paper the equilibrium spin current is calculated for a quasi-2D electron gas with finite thickness under in-plane magnetic field in the presence of Rashba- and Dresselhaus spin-orbit interactions. Note that the problem has been solved for 2D electron gas in Refs. [37–42] in the absence of the magnetic field. Our calculations show that the in-plane magnetic field generates out-of-plane spin current, which appears exclusively due to sub-band

mixing by means of in-plane magnetic field. Although the equilibrium spin current vanishes at  $\alpha = \beta$  in the absence of the magnetic field (see, Refs. [41, 42]), in-plane magnetic field destroys this symmetry, yielding non-zero persistent spin-current at  $\alpha = \beta$ . The magnetic field strongly changes  $\mathbf{J}^{S_x}$  and  $\mathbf{J}^{S_y}$  spin-current components, and contributes new terms to them.

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#### APPENDIX

Equations (18) and (19) are reduced for  $n = 1$  into the following form

$$\begin{aligned} (\tilde{\epsilon} - 3/2) a_1 - c_0 b_1 - \sqrt{2}c_1 b_0 &= 0 \\ (\tilde{\epsilon} - 3/2) b_1 - c_0^* a_1 - \sqrt{2}c_1^* a_0 &= 0 \\ (\tilde{\epsilon} - 1/2) a_0 - c_0 b_0 - \sqrt{2}c_1 b_1 &= 0 \\ (\tilde{\epsilon} - 1/2) b_0 - c_0^* a_0 - \sqrt{2}c_1^* a_1 &= 0, \end{aligned} \quad (79)$$

where  $\tilde{\epsilon} = \tilde{E}/(\hbar\omega)$ , and the higher order terms  $a_2$  and  $b_2$  are neglected, since the levels  $n = 0$  and  $n = 1$  only are filled. Excluding  $a_1$  and  $b_1$  from these equations one gets

$$\left\{ -\frac{c_0^*}{c_1^*}(\tilde{\epsilon} - \frac{3}{2}) - \frac{c_0}{c_1}(\tilde{\epsilon} - \frac{1}{2}) \right\} a_0 + \left\{ \frac{(\tilde{\epsilon} - 3/2)(\tilde{\epsilon} - 1/2)}{c_1^*} + \frac{c_0^2}{c_1} - 2c_1 \right\} b_0 = 0, \quad (80)$$

$$\left\{ \frac{(\tilde{\epsilon} - 3/2)(\tilde{\epsilon} - 1/2)}{c_1} + \frac{c_0^{*2}}{c_1^*} - 2c_1^* \right\} a_0 + \left\{ -\frac{c_0}{c_1}(\tilde{\epsilon} - \frac{3}{2}) - \frac{c_0^*}{c_1^*}(\tilde{\epsilon} - \frac{1}{2}) \right\} b_0 = 0. \quad (81)$$

Eqs. (79) yield a system of equations for  $a_1$  and  $b_1$  too, which differs from Eqs. (80) and (81) by interchanging the coefficients in the front of  $a_0$  in Eq. (80) and of  $b_0$  in Eq. (81). Both system of equations results in the following expression for the energy spectrum

$$[(\tilde{\epsilon} - 3/2)(\tilde{\epsilon} - 1/2) - 2|c_1|^2]^2 - |c_0|^2[(\tilde{\epsilon} - 3/2)^2 + (\tilde{\epsilon} - 1/2)^2] - 2(c_0^{*2}c_1^2 + c_0^2c_1^{*2}) + |c_0|^4 = 0. \quad (82)$$

By using the relation  $(\tilde{\epsilon} - 3/2)^2 + (\tilde{\epsilon} - 1/2)^2 = 2(\tilde{\epsilon} - 3/2)(\tilde{\epsilon} - 1/2) + 1$  in the second term of Eq. (82) one gets a quadratic equation for  $z = (\tilde{\epsilon} - 3/2)(\tilde{\epsilon} - 1/2)$

$$z^2 - 2(|c_0|^2 + 2|c_1|^2)z - |c_0|^2 + |c_0|^4 + 4|c_1|^4 - 2(c_0^{*2}c_1^2 + c_0^2c_1^{*2}) = 0 \quad (83)$$

with the following solutions

$$z = |c_0|^2 + 2|c_1|^2 \pm \sqrt{|c_0|^2 + 2(c_0^*c_1 + c_0c_1^*)^2}. \quad (84)$$

Solution of Eq. (84) for the dimensionless energy  $\tilde{\epsilon}$  is expressed as

$$\tilde{\epsilon}_{\pm}^{(n)} = 1 + \frac{\lambda_n}{2} \sqrt{1 + 4|c_0|^2 + 8|c_1|^2 \mp 4\sqrt{|c_0|^2 + 2(c_0^*c_1 + c_0c_1^*)^2}}, \quad (85)$$

where  $\lambda_n = \pm$  indicates the first ( $\lambda_0 = -$ ) and second ( $\lambda_1 = +$ ) energy subbands, and  $\mp$  assigns two spin-polarized branches in each energy subband. Note that the coefficients  $c_0$  and  $c_1$  are given by Eqs. (11) and (20), furthermore  $c_0$  only depends on the in-plane momentum components  $\{k_x, k_y\}$ .

The coefficients  $a_n$  and  $b_n$  are complex parameters. Solutions of Eqs. (80) and (81) with the normalization condition yield for the modulus  $|a_0| = |b_0|$

$$|a_0|^2 = \frac{(\tilde{\epsilon} - \frac{3}{2})(\tilde{\epsilon} - \frac{1}{2})[(\tilde{\epsilon} - \frac{3}{2})(\tilde{\epsilon} - \frac{1}{2}) - |c_0|^2 - 2|c_1|^2]^2 - |c_0|^2[(\tilde{\epsilon} - \frac{3}{2})(\tilde{\epsilon} - \frac{1}{2}) - 2|c_1|^2]}{4(\tilde{\epsilon} - 1)[(\tilde{\epsilon} - \frac{3}{2})(\tilde{\epsilon} - \frac{1}{2}) - |c_0|^2 - 2|c_1|^2][(\tilde{\epsilon} - \frac{3}{2})(\tilde{\epsilon} - \frac{1}{2}) - (\tilde{\epsilon} - \frac{1}{2})(|c_0|^2 + 2|c_1|^2) + |c_0|^2]}. \quad (86)$$

In the absence of the magnetic field  $B \rightarrow 0$  or  $c_1 \rightarrow 0$  inter-subband coupling disappears, and Eq. (84) is reduced to the form

$$[(\tilde{\epsilon} - 3/2)^2 - |c_0|^2][\tilde{\epsilon} - 1/2]^2 - |c_0|^2 = 0 \quad (87)$$

The expression for  $|a_0|^2$  is simplified as  $|a_0|^2 = 1/2$  under this condition.

The symmetry relations  $a_n = e^{i\theta} b_n^*$  and  $b_n = e^{i\theta} a_n^*$  for the complex coefficients  $a_n = |a_n|e^{i\phi_n^a}$  and  $b_n = |b_n|e^{i\phi_n^b}$  imply that a total phase  $\theta = \phi_n^a - \phi_n^b$  is undefined parameter, whereas the relative phase  $\phi_n^a - \phi_n^b$  for  $n = 0$  and

$n = 1$  can be defined from the relations (80) and (81)

$$\frac{b_n}{a_n} = \pm \exp\{i(\phi_n^b - \phi_n^a)\}, \quad n = 0, 1; \quad (88)$$

$$\phi_0^b - \phi_0^a = \arg \left[ \frac{c_0^*}{c_1^*}(\tilde{\epsilon} - \frac{3}{2}) + \frac{c_0}{c_1}(\tilde{\epsilon} - \frac{1}{2}) \right] - \arg \left[ \frac{(\tilde{\epsilon} - 3/2)(\tilde{\epsilon} - 1/2)}{c_1^*} + \frac{c_0^2}{c_1} - 2c_1 \right]; \quad (89)$$

$$\phi_1^b - \phi_1^a = \arg \left[ \frac{c_0}{c_1}(\tilde{\epsilon} - \frac{3}{2}) + \frac{c_0^*}{c_1^*}(\tilde{\epsilon} - \frac{1}{2}) \right] - \arg \left[ \frac{(\tilde{\epsilon} - 3/2)(\tilde{\epsilon} - 1/2)}{c_1^*} + \frac{c_0^2}{c_1} - 2c_1 \right]. \quad (90)$$

The wave function can be expressed as

$$\Psi(x, y, z) = \exp\{i(k_x x + k_y y) - (z - z_0)^2/2a_B^2\} \sum_{n=0}^{\infty} \frac{H_n((z - z_0)/a_B)}{\sqrt{a_B \sqrt{\pi}} 2^n n!} a_n \begin{pmatrix} 1 \\ \pm e^{i(\phi_n^b - \phi_n^a)} \end{pmatrix}, \quad (91)$$

where the signs  $\pm$  correspond to two different spin-polarized branches.

In order to estimate the equilibrium charge- and spin-currents we have to calculate the mean values of the Pauli matrices  $\langle \sigma_i \rangle_{\mathbf{k}, n_m}$  in the eigenstates given by Eqs. (14) and (15) up to the Fermi level  $n = n_m$ , and integrate the results over  $\{k_x, k_y\}$  components of the in-plane momentum vector,  $-K_{n_m, \pm}^i \leq k_i \leq K_{n_m, \pm}^i$ . Routine calculations yield the following results for  $\langle \sigma_x \rangle_{\mathbf{k}, n_m}$

$$\begin{aligned} \langle \sigma_x \rangle_{\mathbf{k}, n_m} &= \sum_{n=0}^{n_m=1} (a_n^* b_n^*) \sigma_x \begin{pmatrix} a_n \\ b_n \end{pmatrix} = a_0^* b_0 + b_0^* a_0 + a_1^* b_1 + b_1^* a_1 = \\ &= \frac{(c_0^* + c_0) \left[ \left( \tilde{\epsilon} - \frac{3}{2} \right)^2 + \left( \tilde{\epsilon} - \frac{1}{2} \right)^2 - 2|c_0|^2 \right] + 4(c_0^* c_1^2 + c_0 c_1^{*2})}{4(\tilde{\epsilon} - 1) \left[ \left( \tilde{\epsilon} - \frac{3}{2} \right) \left( \tilde{\epsilon} - \frac{1}{2} \right) - |c_0|^2 - 2|c_1|^2 \right]}, \end{aligned} \quad (92)$$

and for  $\langle \sigma_y \rangle_{\mathbf{k}, n_m}$

$$\begin{aligned} \langle \sigma_y \rangle_{\mathbf{k}, n_m} &= \sum_{n=0}^{n_m=1} (a_n^* b_n^*) \sigma_y \begin{pmatrix} a_n \\ b_n \end{pmatrix} = -i(a_0^* b_0 - b_0^* a_0 + a_1^* b_1 - b_1^* a_1) = \\ &= \mp i \lambda_n \frac{(c_0^* - c_0) \left[ \left( \tilde{\epsilon} - \frac{3}{2} \right)^2 + \left( \tilde{\epsilon} - \frac{1}{2} \right)^2 - 2|c_0|^2 \right] - 4(c_0^* c_1^2 - c_0 c_1^{*2})}{4(\tilde{\epsilon} - 1) \left[ \left( \tilde{\epsilon} - \frac{3}{2} \right) \left( \tilde{\epsilon} - \frac{1}{2} \right) - |c_0|^2 - 2|c_1|^2 \right]}. \end{aligned} \quad (93)$$

By neglecting the small terms  $4(c_0^* c_1^2 + c_0 c_1^{*2})$  and  $4(c_0^* c_1^2 - c_0 c_1^{*2})$  in the numerators of Eqs. (92) and (93), correspondingly, and by using the expressions (83) and (85) for the energy spectrum, one gets for  $\langle \sigma_x \rangle_{k_{\pm}, n}$

$$\langle \sigma_x \rangle_{k_{\pm}, n} = \pm \lambda_n \frac{c_0^* + c_0}{2|c_0|}, \quad (94)$$

and for  $\langle \sigma_y \rangle_{k_{\pm}, n}$

$$\langle \sigma_y \rangle_{k_{\pm}, n} = \mp \lambda_n i \frac{c_0^* - c_0}{2|c_0|}. \quad (95)$$

$y$ -components of the spin-current contain in addition an inter-subband coupling terms  $\langle \sigma_i \rangle_{\mathbf{k}, n_m}^{off}$

$$\begin{aligned} \langle \sigma_x \rangle_{\mathbf{k}, n_m}^{off} &= \sum_{n=0}^{n_m=1} \sqrt{2(n+1)} (a_{n+1}^* b_{n+1}^*) \sigma_x \begin{pmatrix} a_n \\ b_n \end{pmatrix} = \sqrt{2} (a_1^* b_0 + b_1^* a_0) = \\ &= \frac{(c_1^* + c_1) \left[ \left( \tilde{\epsilon} - \frac{3}{2} \right) \left( \tilde{\epsilon} - \frac{1}{2} \right) - 2|c_1|^2 \right] + (c_0^{*2} c_1 + c_0^2 c_1^*)}{2(\tilde{\epsilon} - 1) \left[ \left( \tilde{\epsilon} - \frac{3}{2} \right) \left( \tilde{\epsilon} - \frac{1}{2} \right) - |c_0|^2 - 2|c_1|^2 \right]}, \end{aligned} \quad (96)$$

$$\begin{aligned} \langle \sigma_y \rangle_{\mathbf{k}, n_m}^{off} &= \sum_{n=0}^{n_m=1} \sqrt{2(n+1)} (a_{n+1}^* b_{n+1}^*) \sigma_y \begin{pmatrix} a_n \\ b_n \end{pmatrix} = -i(a_1^* b_0 - b_1^* a_0) = \\ &= -i \frac{(c_1^* - c_1) \left[ \left( \tilde{\epsilon} - \frac{3}{2} \right) \left( \tilde{\epsilon} - \frac{1}{2} \right) - 2|c_1|^2 \right] + (c_0^{*2} c_1 - c_0^2 c_1^*)}{2(\tilde{\epsilon} - 1) \left[ \left( \tilde{\epsilon} - \frac{3}{2} \right) \left( \tilde{\epsilon} - \frac{1}{2} \right) - |c_0|^2 - 2|c_1|^2 \right]}. \end{aligned} \quad (97)$$

The transverse component of the spin current  $\langle \mathbf{J}^{S_z} \rangle$  is proportional to  $\langle \sigma_z \rangle_{\mathbf{k}, n_m}$ , which is given by a simple expression

$$\langle \sigma_z \rangle_{\mathbf{k}, n_m} = \sum_{n=0}^{n_m=1} \sqrt{2(n+1)} (a_{n+1}^* b_{n+1}^*) \sigma_z \begin{pmatrix} a_n \\ b_n \end{pmatrix} = \sqrt{2} (a_1^* a_0 - b_1^* b_0) = \frac{(c_0^* c_1 - c_0 c_1^*)}{2(\tilde{\epsilon} - 1) \left[ (\tilde{\epsilon} - \frac{3}{2}) (\tilde{\epsilon} - \frac{1}{2}) - |c_0|^2 - 2|c_1|^2 \right]} = \pm \lambda_n \frac{c_0^* c_1 - c_0 c_1^*}{|c_0|}. \quad (98)$$

The momentum dependent factor  $c_0^* c_1 - c_0 c_1^*$  in  $\langle \sigma_z \rangle_{\mathbf{k}, n_m}$  is given as

$$c_0^* c_1 - c_0 c_1^* = -i \frac{\omega_B}{(\omega \hbar)^2} \sqrt{\frac{m^*}{\hbar}} [(\alpha^2 - \beta^2) k_x - \beta \omega_z]. \quad (99)$$

In order to find the limit of integration in momentum space we transform the momentum components  $\{k_x, k_y\}$

into polar coordinates  $k_x = k \cos \varphi$ ,  $k_y = k \sin \varphi$  and write Eqs. (45) and (46) with fixed Fermi energy as

$$(\kappa^2 - A_0)^2 = A_1 \kappa^2 + A_2 \kappa + A_3, \quad (100)$$

where  $\kappa = k + \frac{e E_g \omega_B}{\hbar \omega^2} \frac{\sin \varphi}{\cos^2 \varphi + (\omega_0^2 / \omega^2) \sin^2 \varphi}$  is a shifted momentum modulus, and

$$A_0 = \frac{[E_F - \frac{\hbar \omega}{2} (n+1)] (\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi) + \frac{e^2 E_g^2}{2 m^* \omega^2}}{\frac{\hbar^2}{2 m^*} (\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi)^2}; \quad (101)$$

$$A_1 = \frac{(\alpha \cos \varphi + \beta \frac{\omega_0^2}{\omega^2} \sin \varphi)^2 + (\beta \cos \varphi + \alpha \frac{\omega_0^2}{\omega^2} \sin \varphi)^2}{\frac{\hbar^4}{4 m^{*2}} (\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi)^2}; \quad (102)$$

$$A_2 = \frac{8 e E_g m^{*2} \omega_B / (\hbar^5 \omega^2) \cos \varphi}{(\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi)^3} \left\{ 2 \alpha \beta (\cos^2 \varphi - \frac{\omega_0^2}{\omega^2} \sin^2 \varphi) - (\alpha^2 + \beta^2) \frac{\omega_B^2}{\omega^2} \sin \varphi \cos \varphi \right\} + \frac{8 \omega_z m^{*2} (\alpha \frac{\omega_0^2}{\omega^2} \sin \varphi + \beta \cos \varphi)}{\hbar^4 (\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi)^2} \quad (103)$$

$$A_3 = \frac{\left( \frac{2 e E_g m^* \omega_B}{\hbar^3 \omega^2} \right)^2 \cos^2 \varphi (\alpha^2 + \beta^2 - 4 \alpha \beta \sin \varphi \cos \varphi)}{(\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi)^4} + \frac{8 e E_g m^{*2} \omega_B \omega_z \cos \varphi (\alpha \cos \varphi - \beta \sin \varphi)}{\hbar^5 \omega^2 (\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi)^4} + \frac{4 m^{*2} \omega_z^2}{\hbar^4 (\cos^2 \varphi + \frac{\omega_0^2}{\omega^2} \sin^2 \varphi)^2}. \quad (104)$$

The parameter  $A_0$  does not depend on small SO coupling constants  $\alpha$  and  $\beta$  and weakly depends on the gate electric field and in-plane magnetic field, whereas  $A_1 \sim O(\alpha^2, \beta^2)$ ,  $A_2 \sim O(E_g \omega_B \alpha^2, E_g \omega_B \beta^2)$ , and  $A_3 \sim O(E_g^2 \omega_B^2 \alpha^2, E_g^2 \omega_B^2 \beta^2)$ . By introducing an unknowing parameter  $y$  Eq. (100) can be rewritten as

$$(\kappa^2 - A_0 + y)^2 = (A_1 + 2y) \kappa^2 + A_2 \kappa + A_3 - 2y A_0 + y^2 = (A_1 + 2y) \left[ \kappa + \frac{A_2}{2(A_1 + 2y)} \right]^2 + R(y). \quad (105)$$

The parameter  $y$  is found under the condition

$$R(y) = 0 \quad \text{or} \quad y^3 + \frac{A_1 - 4A_0}{2} y^2 - (A_0 A_1 - A_3) y + \frac{A_3 A_1 - 4A_2^2}{8} = 0, \quad (106)$$

yielding

$$y = \frac{4A_0 - A_1}{6} + \sqrt[3]{-\frac{Q}{2} \pm \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}}} - \frac{P}{3 \sqrt[3]{-\frac{Q}{2} \pm \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}}}}, \quad (107)$$

where

$$P = -A_0A_1 + A_3 - \frac{(4A_0 - A_1)^2}{12},$$

$$Q = -\frac{(4A_0 - A_1)^3}{108} + \frac{(A_0A_1 - A_3)(4A_0 - A_1)}{6} + \frac{4A_3A_1 - A_2^2}{8}. \quad (108)$$

Expression for  $y$  can be simplified to the form

$$y = 2A_0 - \frac{A_3}{2A_0}. \quad (109)$$

Finally, the Fermi momentum  $k_{n,\pm}^F$  for each sub-band and spin-branch is found from Eq. (105) under the condition  $R(y) = 0$

$$k_{n,\pm}^F = -\frac{eE_g\omega_B}{\hbar\omega^2} \frac{\sin\varphi}{\cos^2\varphi + \frac{\omega_g^2}{\omega^2} \sin^2\varphi} + \sqrt{A_0 + \frac{A_1}{4} - \frac{A_3}{4A_0}} \pm \sqrt{\frac{A_1}{4} + \frac{A_3}{4A_0} + \frac{A_2}{4\sqrt{A_0}}}. \quad (110)$$

The equation (110) with Eqs. (104) yields an evident expression for  $k_{n,\pm}^F$ .

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